

1. [3 points] Consider the sum of three **4-sided dice**. A four-sided die is shaped like a pyramid made of equilateral triangles, and has the numbers 1,2,3,4 on its sides.
- a. Construct the pdf and cdf for the sum of three 4-sided dice. In 64ths:

	3	4	5	6	7	8	9	10	11	12
Pdf	1	3	6	10	12	12	10	6	3	1
Cdf	1	4	10	20	32	44	54	60	63	64

- b. Using the cdf, show the probability of getting a sum in the range 6 to 8: **(34/64)**
2. [3 points] Consider the sum of 3 coin flips, where heads is assigned the value 1 and tails is assigned the value 0.
- a. What is the mean of the sum of 3 coin flips? What proportion of the time would you expect to see the sum of 3 coin flips take on its mean value? **mean=1.5, 0% of the time**

	0	1	2	3
Pdf	1	3	3	1
Cdf	1	4	7	8

- b. What is the variance of the sum of 3 coin flips? **V=0.75**
3. [1 point] Which of the following is a linear regression model:
- a. $Y_i = \alpha + \beta_1 X_i + \beta_2 X_i^2 + \varepsilon_i$
- b. $\log(Y_i) = \beta_0 + \beta_1 \log(X_i) + \varepsilon_i$
- c. $Y_i = \beta_0 + \beta_1 e^{X_i} + \varepsilon_i$
- d. all of the above**
- e. none of the above
4. [2 points] Why does s^2 have $n-1$ in its denominator rather than n ? **So that it is an unbiased estimator of V---they should prove this.**

5. [6 points] Let 2 random variables have the following joint pdf, with X in rows, and Y in columns:

	$Y=-1$	$Y=0$	$Y=1$
$X=-1$	0.125	0.15	0.125
$X=0$	0	0.2	0
$X=1$	0.125	0.15	0.125

- a. What is the conditional mean of Y for each value of X ? **$E[Y|x]=g(x)=0$**
- b. What is the covariance of X and Y ? Are X and Y independent? **Cov(X,Y)=0, not independent (eg, V(X) depends on Y)**
- c. What is the probability that $X=1$? **0.40**
- d. Assume the even simpler regression model $Y_i = \beta X_i + \varepsilon_i$. Given a large number of observations from this joint pdf, what would you expect the estimated coefficient to be? **0--they could work this out by weighting the formula for beta-hat, or argue it from the conditional mean not depending on X.**
6. [2 points] Prove that the sample mean is an unbiased estimator of the population mean. Be sure to write down all your assumptions. **standard proof in my notes, or any other**
7. [2 points] Suppose that Z is the average of n iid observations of a random variable X . Suppose that $V(X)=1$. Suppose that n is very large. Construct a standard normally distributed test statistic for the hypothesis that X is drawn from a distribution whose mean is 2. **$(Z-2)/(1/n^{1/2}) \sim N(0,1)$**

8. [2 points] Consider the even simpler regression model $Y_i = \beta X_i + \varepsilon_i$, where Y earnings in thousands of dollars and X is age in years. Say that your estimated value of the coefficient is 4, and that the variance of this estimated coefficient is 4.
- What is estimated difference in earnings between a person who is 25 and a person who is 40?
\$60,000
 - How would you use the tables in the back of your textbook to test the hypothesis that the slope of earnings with respect to age is zero? **find the p-value for a standard normal equal to $4/2=2$. they should state whether it is one- or two-sided.**
9. [2 points] Suppose that $X \sim N(0,1)$, $U \sim N(0,1)$ and $Y=2X+U$.
- What is the conditional expectation of Y given that $X=2$? **4**
 - What is the distribution of $Z=X^2+U^2$? **chi-square with 2 df**
10. [2 points] Suppose that $X \sim N(2,2)$ and $Y=2X+5$.
- What is the mean of Y ? What is the variance of Y ? **$E[Y]=9$; $V[Y]=8$; $Y \sim N(9,8)$**
What is the distribution of Y ?